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Spectroscopic Capabilities of $LaF_3:Er^{3+}$ Crystals for MIR Lasers Cascade Operation

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Abstract—A theoretical study of the spectroscopic properties of the low-phonon $LaF_3:Er^{3+}$ crystals determining their laser capabilities in the mid-infrared wavelength range (2.62–4.97 μm) was carried out. The wave functions of the Stark sublevels of $^4S_{3/2}$, $^4F_{9/2}$, $^4I_{9/2}$, $^4I_{11/2}$, and $^4I_{13/2}$ of Er^{3+} ion are constructed by the LSJM-representation. The line strengths of the indirect electric dipole and magnetic dipole inter-Stark transitions are computed and the main spectroscopic and kinetic characteristics of the optical spectrum of the impurity ion are determined.

Keywords: Stevens coefficients, line strength, inter-Stark transitions, cross-sections

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1. INTRODUCTION

Lanthanum fluoride crystals doped with the rare earth ions ($LaF_3:Er^{3+}$), having a low-frequency phonon spectrum ($\omega_m \cong 300–350\,\text{cm}^{-1}$), are one of the promising laser materials for obtaining the lasing in the mid-infrared (MIR) wavelength region, [1]. A targeted search for materials for MIR lasers is of particular practical importance because they are an integral part of an optical LIDAR system. The latter can be used to collect important data on the chemical composition of the atmosphere: to identify the concentrations of toxic compounds CO_2, CH_4, CO, NO_2, SO_2, and etc. [2]. The optical spectra of impurity absorption and emission of the $LaF_3:Er^{3+}$ crystal were studied in [3–6]. In particular, the energy levels of the impurity ion were determined in [4] (Fig. 1), the standard analysis of the Judd–Ofelt absorption spectra was carried out, and the intensity parameters ($\Omega_2 = 1.07\times10^{-20}\text{cm}^2$, $\Omega_4 = 0.28\times10^{-20}\text{cm}^2$, and $\Omega_6 = 0.63\times10^{-20}\text{cm}^2$) were determined [3–6]. In [3], the probabilities of indirect electric dipole (IED) and magnetic dipole (MD) intermultiplet transitions were computed, and the lifetimes of excited multiplets were determined.

In this work, we present the results of quantitative computations of the main spectroscopic characteristics of the $LaF_3:Er^{3+}$crystal, which determine their laser potential in the wavelength range 2.62–4.97 μm. The computations were performed taking into account the Stark structure of the optical spectrum of the impurity ion.

2. THE WAVE FUNCTIONS OF THE STARK SUBLEVELS

As is known, the degenerate multiplet states in a crystal field (CF) of a free ion are split into the Kramers doublets, while the wave function of the ν-th Stark state, in the weak CF approximation (the LSJM representation), is constructed as a linear combination

$$|\nu\rangle = \sum_M a_{JM}^{(\nu)}\,|LSJM\rangle, \tag{1}$$

where L and S are the angular and spin moments, M is the projection of the total angular momentum J, and $a_{JM}^{(\nu)}$ are the numerical coefficients. Assuming that the point symmetry of the nearest environment of

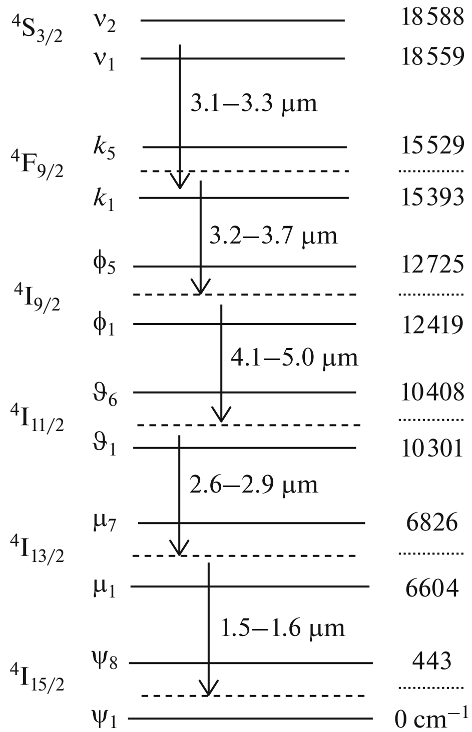

Fig. 1. The energy level diagram of LaF_3–Er^{3+}.

the impurity ion is D_{3h} in the crystal matrix $LaF_3:Er^{3+}$ [6], the Hamiltonian CF, in the framework of the point charge approximation, can be written in the form:

$$\widehat{H}_{CF} = \alpha_J B_{20}\widehat{O}_{20} + \beta_J B_{40}\widehat{O}_{40} + \gamma_J B_{60}\widehat{O}_{60} + \gamma_J B_{60}\widehat{O}_{60}, \tag{2}$$

where α_J, β_J and γ_J are the Stevens constants, the values of which for the Er^{3+} ion are given in Appendix [8], and B_{kq} are the CF parameters, the numerical values of which, determined from the condition of the best agreement between the computed and experimental values of the Stark splittings of multiplet states, are given in в [6]: $B_{20} = 141$, $B_{40} = 145$, $B_{60} = 48.3$, $B_{66} = 430$ (in cm^{-1}), and $\widehat{O}_{kq}$ are the equivalent operators whose matrix elements are tabulated in [9]. The wave functions of Stark states, constructed by the intramultiplet diagonalization of the potential CF (2) based on the basis functions of irreducible representations of the point group D_{3h} [7], are given in the Appendix.

3. THE SPECTROSCOPIC CHARACTERISTICS OF THE $LaF_3:Er^{3+}$ CRYSTAL

In a theoretical study of the spectroscopic properties of impurity crystals, the most convenient value is the line strength of the inter-Stark transition $i \to f$ [10]:

$$S_{i\to f} = \chi_{ed} S^{ed}_{i\to f} + \chi_{md} S^{md}_{i\to f}, \tag{3}$$

where the first term corresponds to the IED

$$S^{(ied)}_{i\to f} = \sum_{t=2,4,6} \Omega_t A^{(ied)}_t (i \to f) \left|\langle f \| U_t \| i\rangle\right|^2, \tag{4}$$

and the second to the MD transitions

$$S^{(md)}_{i\to f} = A^{(md)}_{i\to f} S_{md}. \tag{5}$$

Table 1. The spectroscopic characteristics of spectral lines of the $\nu\left({}^4S_{3/2}\right) \to k\left({}^4F_{9/2}\right)$ inter-Stark transitions

transition	λ, μm	$S_{i\to f}$, $10^{-23}\,\text{cm}^2$	$^*\sigma\Gamma_{\text{cm}^{-1}}$, $10^{-20}\,\text{cm}^2$	$A_{i\to f}$, s^{-1}
$\nu_1 \to k_1$	3.159	3.6238	0.2809	0.083
$\to k_2$	3.205	3.2645	0.2437	0.072
$\to k_3$	3.213	1.3330	0.1015	0.029
$\to k_4$	3.244	2.5503	0.1928	0.054
$\to k_5$	3.300	2.5743	0.1921	0.052
$\nu_2 \to k_1$	3.130	1.6964	0.1329	0.040
$\to k_2$	3.176	2.0557	0.1574	0.046
$\to k_3$	3.184	3.9857	0.3060	0.089
$\to k_4$	3.213	2.7695	0.2063	0.060
$\to k_5$	3.269	2.8571	0.2139	0.059

* Γ is the width of the corresponding spectral line in cm^{-1}.

In (3)–(5) the following notation was introduced: $\chi_{ed} = n\left(n^2+2\right)^2/9$ and $\chi_{md} = n^3$ are the corrections of the local CF (n is the refractive index at the transition length), $\Omega_t\left(t = 2, 4, 6\right)$ are the intensity parameters, $\langle i\|u_t\| f\rangle$ is the reduced matrix element of the irreducible unit operator u_t of rank t, the explicit expression of which is given in the works [10, 11], $A_t^{(ied)}(i \to f)$ and $A_{i\to f}^{(md)}$ are the coefficients of the inter-Stark IED and MD transitions [11–13]:

$$A_t^{(ied)}(i \to f) = \sum_{m=-t}^{t}\left|\sum_{M_i,M_f}(-1)^{J_f-M_f}\,a_{J_f,M_f}^{*(f)}a_{J_i,M_i}^{(i)}\begin{pmatrix} J_f & t & J_i \\ -M_f & m & M_i\end{pmatrix}\right|^2, \tag{6}$$

$$A_{i\to f}^{(md)} = \frac{1}{2J_f+1}\sum_{m}\left|\sum_{M_iM_f}(-1)^{J_f-M_f}\,a_{J_fM_f}^{*(f)}a_{J_iM_i}^{(i)}\begin{pmatrix} J_f & 1 & J_i \\ -M_f & m & M_i\end{pmatrix}\right|^2, \tag{7}$$

S_{md} is the line strength of the inter-multiplet MD transition [3, 11], $\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot\end{pmatrix}$ is the $3j$ symbol.

The reduced matrix elements were calculated using the values of the genealogical coefficients ($4f^{11}$) of the ion Er^{3+} calculated in [8]:

$$\left|\left\langle {}^4I_{11/2}\|u_2\|{}^4I_{13/2}\right\rangle\right|^2 = 0.0332,\quad \left|\left\langle {}^4I_{11/2}\|u_4\|{}^4I_{13/2}\right\rangle\right|^2 = 0.1706,\quad \left|\left\langle {}^4I_{11/2}\|u_6\|{}^4I_{13/2}\right\rangle\right|^2 = 1.0915, \tag{8a}$$

$$\left|\left\langle {}^4I_{9/2}\|u_2\|{}^4I_{11/2}\right\rangle\right|^2 = 0.0021,\quad \left|\left\langle {}^4I_{9/2}\|u_4\|{}^4I_{11/2}\right\rangle\right|^2 = 0.0690,\quad \left|\left\langle {}^4I_{9/2}\|u_6\|{}^4I_{11/2}\right\rangle\right|^2 = 0.1520, \tag{8b}$$

$$\left|\left\langle {}^4F_{9/2}\|u_2\|{}^4I_{9/2}\right\rangle\right|^2 = 0.122,\quad \left|\left\langle {}^4F_{9/2}\|u_4\|{}^4I_{9/2}\right\rangle\right|^2 = 0.0061,\quad \left|\left\langle {}^4F_{9/2}\|u_6\|{}^4I_{9/2}\right\rangle\right|^2 = 0.0203, \tag{8c}$$

$$\left|\left\langle {}^4S_{3/2}\|u_2\|{}^4F_{9/2}\right\rangle\right|^2 = 0,\quad \left|\left\langle {}^4S_{3/2}\|u_4\|{}^4F_{9/2}\right\rangle\right|^2 = 0.0001,\quad \left|\left\langle {}^4S_{3/2}\|u_6\|{}^4F_{9/2}\right\rangle\right|^2 = 0.0228. \tag{8d}$$

The values of line strengths, cross-sections, and probabilities of spontaneous inter-Stark transitions $\nu_i\left({}^4S_{3/2}\right) \to k_f\left({}^4F_{9/2}\right)$, $k_i\left({}^4F_{9/2}\right) \to \phi_f\left({}^4I_{9/2}\right)$, $\phi_i\left({}^4I_{9/2}\right) \to \vartheta_f\left({}^4I_{11/2}\right)$ and $\vartheta_i\left({}^4I_{11/2}\right) \to \mu_f\left({}^4I_{13/2}\right)$, computed at $n = 1.6$ are given in Tables 1–4.

Table 1 shows that the force of the intermultiplet transition ${}^4S_{3/2} \to {}^4F_{9/2}$ is small $\left(5.34\times10^{-22}\ \text{cm}^2\right)$, which explains the negligible contribution of this transition to the process of decay of the level ${}^4S_{3/2}$ [3].

Table 2. The spectroscopic characteristics of spectral lines of the $k\left({}^4F_{9/2}\right) \to \phi\left({}^4I_{9/2}\right)$ inter-Stark transitions

Transition	λ, μm	$S_{i\to f}$, 10^{-23} cm^2	*$\sigma\Gamma_{cm^{-1}}$, 10^{-20} cm^2	$A_{i\to f}$, s^{-1}
$k_1 \to \phi_1$	3.362	11.8418	0.7963	0.2256
$\to \phi_2$	3.499	19.7870	1.2779	0.3335
$\to \phi_3$	3.587	12.1305	0.7259	0.1894
$\to \phi_4$	3.720	4.5166	0.2974	0.0688
$\to \phi_5$	3.748	4.9003	0.2988	0.0679
$k_2 \to \phi_1$	3.311	9.6590	0.6594	0.1925
$\to \phi_2$	3.444	3.8915	0.2558	0.0691
$\to \phi_3$	3.529	21.2723	1.2935	0.3494
$\to \phi_4$	3.658	13.1575	1.4646	0.1939
$\to \phi_5$	3.685	5.6359	0.3461	0.0817
$k_3 \to \phi_1$	3.303	10.3476	0.7062	0.2062
$\to \phi_2$	3.434	7.1027	0.4679	0.1272
$\to \phi_3$	3.519	34.6767	2.2264	0.5745
$\to \phi_4$	3.647	12.7538	0.7900	0.1895
$\to \phi_5$	3.674	19.9460	1.2285	0.2916
$k_4 \to \phi_1$	3.271	15.7104	1.0853	0.3247
$\to \phi_2$	3.400	8.9743	0.5966	0.1650
$\to \phi_3$	3.483	8.6124	0.5593	0.1477
$\to \phi_4$	3.609	20.6727	1.2947	0.3175
$\to \phi_5$	3.635	13.7814	0.8558	0.2065
$k_5 \to \phi_1$	3.215	15.9011	1.1155	0.3442
$\to \phi_2$	3.340	8.7884	0.5946	0.1704
$\to \phi_3$	3.420	8.1437	0.5381	0.1471
$\to \phi_4$	3.541	20.7252	1.3207	0.3376
$\to \phi_5$	3.566	14.2210	0.9006	0.2258

* Γ is the width of the corresponding spectral line in cm^{-1}.

The strength of the IED intermultiplet transition ${}^4F_{9/2} \to {}^4I_{9/2}$ is 6.54×10^{-21} cm^2, which for the probability of spontaneous radiative transition results to the value of 1.1 s^{-1} (Table 2) and is consistent with the value obtained in [3]: $A_{ed} = 1.2$ s^{-1}.

Note that in [3] the multiplet states mixed in the orbital and spin moments with a fixed value of the total angular momentum were considered. As a result, the ban on the magnetic dipole transition ${}^4F_{9/2} \to {}^4I_{9/2}$ is lifted, which results in an increase in the probability of the total radiative transition up to $A_{\text{tot}} = 3.6$ s^{-1}. Note that a similar discrepancy is possible in all cases when intermultiplet magnetic dipole transitions are forbidden. In the absence of a ban on magnetic dipole transitions, as in the case of a ${}^4I_{11/2} \to {}^4I_{13/2}$ transition, there is good agreement between the calculated value of the probability of the radiative transition (Table 4), with the value obtained in [3], equal to 16.1 s^{-1}.

4. CONCLUSION

The obtained values of the cross-sections and the probabilities of spontaneous inter-Stark transitions make it possible to reveal the lasing capabilities (including the conditions for effective cascade generation)

Table 3. The spectroscopic characteristics of spectral lines of the $\phi\left({}^4I_{9/2}\right) \to \vartheta\left({}^4I_{11/2}\right)$ inter-Stark transitions

Transition	λ, μm	$S_{i\to f}$, 10^{-23} cm^2	*$\sigma\Gamma_{\mathrm{cm}^{-1}}$, 10^{-20} cm^2	$A_{i\to f}$, s^{-1}
$\phi_1 \to \vartheta_1$	4.721	66.2793	3.4488	0.4555
$\to \vartheta_2$	4.744	166.4830	8.6254	1.1296
$\to \vartheta_3$	4.787	4.3895	0.2254	0.0289
$\to \vartheta_4$	4.819	42.0707	2.1483	0.2732
$\to \vartheta_5$	4.852	9.4657	0.4797	0.0600
$\to \vartheta_6$	4.973	4.2481	0.2099	0.0250
$\phi_2 \to \vartheta_1$	4.476	26.0803	1.4294	0.2096
$\to \vartheta_2$	4.596	24.0186	1.3135	0.1909
$\to \vartheta_3$	4.535	44.4508	2.4091	0.3455
$\to \vartheta_4$	4.564	82.3669	4.4352	0.6277
$\to \vartheta_5$	4.593	90.2827	4.8296	0.6746
$\to \vartheta_6$	4.701	26.9390	1.4076	0.1875
$\phi_3 \to \vartheta_1$	4.340	7.8409	0.4434	0.0693
$\to \vartheta_2$	4.359	5.0287	0.2829	0.0438
$\to \vartheta_3$	4.396	185.8530	10.3694	1.5765
$\to \vartheta_4$	4.423	3.1428	0.1746	0.0263
$\to \vartheta_5$	4.450	83.7593	4.6209	0.6868
$\to \vartheta_6$	4.552	8.4148	0.4544	0.0646
$\phi_4 \to \vartheta_1$	4.160	120.5810	7.1160	1.2103
$\to \vartheta_2$	4.177	37.1515	2.1817	0.3676
$\to \vartheta_3$	4.211	3.5114	0.2047	0.0340
$\to \vartheta_4$	4.235	63.8633	3.6971	0.6054
$\to \vartheta_5$	4.261	22.2426	1.2817	0.2079
$\to \vartheta_6$	4.354	2.4183	0.1363	0.0212
$\phi_5 \to \vartheta_1$	4.125	27.6117	1.6407	0.2832
$\to \vartheta_2$	4.143	4.5983	0.2724	0.0468
$\to \vartheta_3$	4.175	5.9414	0.3497	0.0592
$\to \vartheta_4$	4.200	22.0277	1.2871	0.2148
$\to \vartheta_5$	4.225	31.3596	1.8238	0.3015
$\to \vartheta_6$	4.316	202.6050	11.5128	1.8159

* Γ is the width of the corresponding spectral line in cm^{-1}.

of the LaF_3:Er^{3+} crystal. So, when excited at a wavelength of 0.52 μm, the quasi-three-level lasing is possible according to the scheme below

$$ {}^4I_{15/2} \overset{0.52\ \mu\mathrm{m}}{\Rightarrow} {}^2H_{11/2} \rightsquigarrow \nu_{1,2}\left({}^4S_{3/2}\right) \overset{3.213\ \mu\mathrm{m}}{\rightarrow} k_{3,4}\left({}^4F_{9/2}\right) \rightsquigarrow k_1\left({}^4F_{9/2}\right) \overset{3.50\ \mu\mathrm{m}}{\rightarrow} \phi_2\left({}^4I_{9/2}\right) \rightsquigarrow, $$

$$ \rightsquigarrow \phi_1\left({}^4I_{9/2}\right) \overset{4.74\ \mu\mathrm{m}}{\rightarrow} \vartheta_2\left({}^4I_{11/2}\right) \rightsquigarrow \vartheta_{1,2}\left({}^4I_{11/2}\right) \overset{2.716-2.742\ \mu\mathrm{m}}{\rightarrow} \mu_2\left({}^4I_{13/2}\right) \rightsquigarrow \mu_1\left({}^4I_{13/2}\right), $$

or its separate fragment.

Table 4. The spectroscopic characteristics of spectral lines of the $\vartheta\left({}^4I_{11/2}\right) \to \mu\left({}^4I_{13/2}\right)$ inter-Stark transitions

transition	λ, μm	$S_{i\to f}$, 10^{-23} cm^2	*$\sigma\Gamma_{cm^{-1}}$, 10^{-20} cm^2	$A_{i\to f}$, s^{-1}
$\vartheta_1 \to \mu_1$	2.705	243.1820	20.4526	8.9275
$\to \mu_2$	2.724	104.0750	8.6588	3.7371
$\to \mu_3$	2.754	50.3983	4.1065	1.7511
$\to \mu_4$	2.777	22.7269	1.8401	0.7644
$\to \mu_5$	2.795	21.3236	1.7108	0.7095
$\to \mu_6$	2.819	3.8441	0.3027	0.1239
$\to \mu_7$	2.878	21.9667	1.6920	0.6645
$\vartheta_2 \to \mu_1$	2.698	19.2238	1.5785	0.7057
$\to \mu_2$	2.717	162.9140	13.5351	5.8498
$\to \mu_3$	2.747	12.9032	1.0395	0.4483
$\to \mu_4$	2.769	178.6010	14.5659	6.0720
$\to \mu_5$	2.787	38.4929	3.0897	1.2807
$\to \mu_6$	2.811	27.0332	2.1520	0.8804
$\to \mu_7$	2.869	28.3668	2.1938	0.8671
$\vartheta_3 \to \mu_1$	2.684	75.0195	6.2943	2.8162
$\to \mu_2$	2.703	40.3716	3.3758	1.4821
$\to \mu_3$	2.732	212.8830	17.6765	7.5604
$\to \mu_4$	2.755	26.2279	2.1255	0.9113
$\to \mu_5$	2.772	84.4569	6.9061	2.8713
$\to \mu_6$	2.796	18.5120	1.4647	0.6094
$\to \mu_7$	2.854	10.0976	0.7931	0.3152
$\vartheta_4 \to \mu_1$	2.674	4.4131	0.3682	0.1675
$\to \mu_2$	2.693	12.8300	1.0699	0.4763
$\to \mu_3$	2.722	24.0152	1.9632	0.8623
$\to \mu_4$	2.744	89.8604	7.4216	3.1565
$\to \mu_5$	2.762	72.3305	5.9300	2.4859
$\to \mu_6$	2.786	229.2190	18.5289	7.6265
$\to \mu_7$	2.843	34.8710	2.7329	1.0002
$\vartheta_5 \to \mu_1$	2.664	7.6162	0.6385	0.2924
$\to \mu_2$	2.682	54.2900	4.5359	2.0380
$\to \mu_3$	2.712	77.5617	6.4740	2.8160
$\to \mu_4$	2.734	57.1257	4.7432	2.0288
$\to \mu_5$	2.751	160.1360	13.1543	5.5639
$\to \mu_6$	2.775	89.4245	7.3273	3.0402
$\to \mu_7$	2.831	21.3726	1.6796	0.6814
$\vartheta_6 \to \mu_1$	2.629	51.4941	4.2540	1.9770
$\to \mu_2$	2.647	26.2572	2.1238	0.9857
$\to \mu_3$	2.675	23.0353	1.8611	0.8363
$\to \mu_4$	2.697	26.2061	2.0799	0.9307
$\to \mu_5$	2.714	23.9994	1.8930	0.8339
$\to \mu_6$	2.737	32.7152	2.5604	1.1122
$\to \mu_7$	2.792	284.0730	22.1554	9.0565

* Γ is the width of the corresponding spectral line in cm^{-1}.

Appendix

Table 5. The Stevens coefficients of the lower Er^{3+} ion multiplets

$^{2S+1}L_J$	J	α_J	β_J	γ_J
^4I_J	$\frac{15}{2}$	$\frac{2^2}{3^2\times5^2\times7}$	$\frac{2}{3^2\times5\times7\times11\times13}$	$\frac{2^3}{3^3\times7\times11^2\times13^2}$
	$\frac{13}{2}$	$\frac{1}{5^2\times13}$	$\frac{2^2}{3^2\times5\times11^2\times13}$	$\frac{1}{3^3\times11^2\times13^2}$
	$\frac{11}{2}$	$\frac{2^3\times17}{3\times7\times11^2\times13}$	$\frac{2\times17\times47}{3^3\times5\times7\times11^3\times13}$	$\frac{2^5\times5\times19}{3^4\times7\times11^3\times13^2}$
	$\frac{9}{2}$	$\frac{7}{3^2\times11^2}$	$\frac{2^3\times17}{3^3\times11^3\times13}$	$\frac{5\times17\times19}{3^3\times7\times11^3\times13^2}$
^4F_J	$\frac{9}{2}$	$\frac{157}{2^5\times3^4\times7}$	$-\frac{1}{2^2\times3\times7^2\times11}$	$\frac{163}{2^4\times3^5\times7^2\times11\times13}$

Table 6. The wave functions of Stark states of Er^{3+} ion in LaF_3

$^4S_{3/2}$:	$\nu_2 = \left\|\frac{3}{2}\pm\frac{3}{2}\right\rangle$
	$\nu_1 = \pm\left\|\frac{3}{2}\pm\frac{1}{2}\right\rangle$
$^4F_{9/2}$:	$k_5 = 0.7178\left\|\frac{9}{2}\mp\frac{7}{2}\right\rangle + 0.6962\left\|\frac{9}{2}\pm\frac{5}{2}\right\rangle$
	$k_4 = -0.6962\left\|\frac{9}{2}\mp\frac{7}{2}\right\rangle + 0.7178\left\|\frac{9}{2}\pm\frac{5}{2}\right\rangle$
	$k_3 = \pm0.9723\left\|\frac{9}{2}\mp\frac{9}{2}\right\rangle \pm 0.2338\left\|\frac{9}{2}\pm\frac{3}{2}\right\rangle$
	$k_2 = \mp0.2338\left\|\frac{9}{2}\mp\frac{9}{2}\right\rangle \pm 0.9723\left\|\frac{9}{2}\pm\frac{3}{2}\right\rangle$
	$k_1 = \left\|\frac{9}{2}\pm\frac{1}{2}\right\rangle$
$^4I_{9/2}$:	$\phi_5 = \pm0.9332\left\|\frac{9}{2}\mp\frac{9}{2}\right\rangle \pm 0.3595\left\|\frac{9}{2}\pm\frac{3}{2}\right\rangle$
	$\phi_4 = 0.8487\left\|\frac{9}{2}\mp\frac{7}{2}\right\rangle + 0.5288\left\|\frac{9}{2}\pm\frac{5}{2}\right\rangle$
	$\phi_3 = \left\|\frac{9}{2}\pm\frac{1}{2}\right\rangle$
	$\phi_2 = \mp0.3595\left\|\frac{9}{2}\mp\frac{9}{2}\right\rangle \pm 0.9332\left\|\frac{9}{2}\pm\frac{3}{2}\right\rangle$
	$\phi_1 = -0.5288\left\|\frac{9}{2}\mp\frac{7}{2}\right\rangle + 0.8487\left\|\frac{9}{2}\pm\frac{5}{2}\right\rangle$
$^4I_{11/2}$:	$\vartheta_6 = \pm0.9947\left\|\frac{11}{2}\mp\frac{11}{2}\right\rangle \pm 0.1032\left\|\frac{11}{2}\pm\frac{1}{2}\right\rangle$
	$\vartheta_5 = 0.2235\left\|\frac{11}{2}\mp\frac{9}{2}\right\rangle + 0.9747\left\|\frac{11}{2}\pm\frac{3}{2}\right\rangle$
	$\vartheta_4 = \pm0.4153\left\|\frac{11}{2}\mp\frac{7}{2}\right\rangle \pm 0.9097\left\|\frac{11}{2}\pm\frac{5}{2}\right\rangle$
	$\vartheta_3 = \mp0.1032\left\|\frac{11}{2}\mp\frac{11}{2}\right\rangle \pm 0.9947\left\|\frac{11}{2}\pm\frac{1}{2}\right\rangle$

	$\vartheta_2 = \pm 0.9097\left\|\frac{11}{2} \mp \frac{7}{2}\right\rangle \mp 0.4153\left\|\frac{11}{2} \pm \frac{5}{2}\right\rangle$
	$\vartheta_1 = 0.9747\left\|\frac{11}{2} \mp \frac{9}{2}\right\rangle - 0.2235\left\|\frac{11}{2} \pm \frac{3}{2}\right\rangle$
$^4I_{13/2}$	$\mu_7 = 0.0146\left\|\frac{13}{2} \mp \frac{11}{2}\right\rangle + 0.1063\left\|\frac{13}{2} \pm \frac{1}{2}\right\rangle + 0.9942\left\|\frac{13}{2} \pm \frac{13}{2}\right\rangle$
	$\mu_6 = 0.6695\left\|\frac{13}{2} \mp \frac{7}{2}\right\rangle + 0.7428\left\|\frac{13}{2} \pm \frac{5}{2}\right\rangle$
	$\mu_5 = \pm 0.5067\left\|\frac{13}{2} \mp \frac{9}{2}\right\rangle \pm 0.8621\left\|\frac{13}{2} \pm \frac{3}{2}\right\rangle$
	$\mu_4 = 0.7428\left\|\frac{13}{2} \mp \frac{7}{2}\right\rangle - 0.6695\left\|\frac{13}{2} \pm \frac{5}{2}\right\rangle$
	$\mu_3 = -0.3538\left\|\frac{13}{2} \mp \frac{11}{2}\right\rangle - 0.9295\left\|\frac{13}{2} \pm \frac{1}{2}\right\rangle + 0.1046\left\|\frac{13}{2} \pm \frac{13}{2}\right\rangle$
	$\mu_2 = \pm 0.8621\left\|\frac{13}{2} \mp \frac{9}{2}\right\rangle \mp 0.5067\left\|\frac{13}{2} \pm \frac{3}{2}\right\rangle$
	$\mu_1 = -0.9352\left\|\frac{13}{2} \mp \frac{11}{2}\right\rangle + 0.3532\left\|\frac{13}{2} \pm \frac{1}{2}\right\rangle - 0.0240\left\|\frac{13}{2} \pm \frac{13}{2}\right\rangle$

CONFLICT OF INTEREST

The authors declare no conflict of interest.

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